Compute the first few partial sums of the series

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$$

& discuss convergence.

Compute the first few partial sums of the series

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$$

& discuss convergence.

Partial sum: $S_N = \sum_{n=0}^{N} a_n$ / $S_n = \sum_{n=1}^{N} a_n$

$$S_1 = \frac{1}{1(1+1)} = \frac{1}{2}$$

$$S_2 = \frac{1}{2} + \frac{1}{2(2+1)} = \frac{2}{3}$$

$$S_3 = \frac{2}{3} + \frac{1}{3(3+1)} = \frac{3}{4}$$

$$S_4 = \frac{3}{4} + \frac{1}{4(4+1)} = \frac{4}{5}$$

$$S_N = S_{N-1} + \frac{1}{N(N+1)}$$

$$= S_{N-1} + \frac{1}{N} - \frac{1}{N+1}$$

If $S_{N-1} = \frac{N-1}{N}$

$$S_N = \frac{N-1}{N} + \frac{1}{N} - \frac{1}{N+1}$$

$$= 1 - \frac{1}{N+1}$$

$$= \frac{N}{N+1}$$

$$\therefore \lim_{N\to\infty} S_N = \sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \lim_{N\to\infty} \frac{N}{N+1} = 1$$

$\therefore \sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ converges to 1.